Micro C Exam February 2012
Suggested Solutions

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1. (a) Find all pure and mixed Nash equilibria in the following game:

	t_1	t_2
s_1	$5, 0$	$10, 1$
s_2	$7, 1$	$9, 0$

Solution: Pure strategy NE:

$$(s_2, t_1) \text{ and } (s_1, t_2).$$

In a mixed NE, each player must be indifferent between the pure strategies that he chooses with positive probability. Let p denote the probability that player 1 plays s_1 and let q denote the probability that player 2 plays t_1. Then we have:

$$\begin{aligned} 5q + 10(1-q) &= 7q + 9(1-q); \\ 1 - p &= p. \end{aligned}$$

From these two equations we get

$$p = \frac{1}{2} \text{ and } q = \frac{1}{3}.$$

(b) Consider the following two stage game with two players (1 and 2). In the second stage, the players play one of the following two games (player 1 chooses U or D, player 2 chooses L or R):

Game A:

	L	R
U	$3, 1$	$0, 0$
D	$0, 0$	$1, 3$

Game B:		L	R
	U	$2,2$	$7,0$
	D	$0,6$	$5,5$

In the first stage, player 1 chooses among the actions A and B. If he chooses A then the players play Game A in stage two. If he chooses B then they play Game B in stage two. Player 2 observes the choice of player 1 before stage two.

i. Draw a game tree representing the two stage game.

   Solution: See the game tree on the final page. It is, of course, also perfectly fine to have player 2 moving first in stage 2.

ii. Is it a game of perfect or imperfect information? How many subgames are there in the game (excluding the game itself)?

   Solution: Since the two players move simultaneously in stage two it is a game of imperfect information (in the game tree there are two information sets with two decision nodes). There are two subgames. One subgame starts after player 1 has chosen A in stage one, the other subgame starts after player 1 has chosen B in stage one.

iii. What are the strategies for the two players?

   Solution:

$$S_1 = \{AUU, AUD, ADU, ADD, BUU, BUD, BDU, BDD\}$$

   (the first action is the action in stage one, the second action is the action in game A, the final action is the action in game B).

$$S_2 = \{LL, LR, RL, RR\}$$

   (the first action is the action in game A, the second action is the action in game B).

iv. Find all pure strategy subgame perfect Nash equilibria.

   Solution: In game A there are two pure strategy NE: (U, L) and (D, R). In game B there is only one pure strategy NE: (U, L). Thus we get two SPNE in the two stage game:

$$(AUU, LL) \text{ and } (BDU, RL).$$

   In the first SPNE player 1 and 2 end up with the payoffs 3 and 1, in the second they end up with the payoffs 2 and 2.

2. Two profit maximizing firms (1 and 2) sell differentiated goods. The firms set their prices (p_1 and p_2) simultaneously and independently. The demand function facing firm i is:

$$q_i(p_i, p_j) = a + b \cdot (p_j - p_i),$$

where $a, b > 0$ are constants (do not worry about negative demand in this exercise). The marginal cost for firm i is $c_i \in (0, \frac{a}{b})$. There are no fixed costs.

(a) Write down the profit functions for the two firms. Find the best response functions. Comment on the role of b.

Solution: Profit function for firm i:

$$\pi_i(p_i, p_j) = q_i(p_i, p_j)p_i - c_i q_i(p_i, p_j) = (a + b \cdot (p_j - p_i))(p_i - c_i).$$

Firm i solves:

$$\max_{p_i \geq 0} \pi_i(p_i, p_j).$$

Find the FOC for this problem and solve for p_i to get firm i's best response function:

$$p_i = \frac{a + bp_j + bc_i}{2b}.$$

The higher b is, the closer substitutes are the products of the two firms (when b is higher, consumers respond more to price differences between the products).

(b) Assume the two firms have identical marginal costs:

$$c_1 = c_2 = c.$$

Find the Nash equilibrium [*hint:* the equilibrium is symmetric, the two firms set the same price]. How does the equilibrium price depend on b? Give an intuitive explanation.

Solution: Using the best response functions from above, we get the following conditions for a Nash Equilibrium:

$$p_1^* = \frac{a + bp_2^* + bc}{2b} \text{ and } p_2^* = \frac{a + bp_1^* + bc}{2b}.$$

Solve these two equations to get:

$$p_1^* = p_2^* = \frac{a + bc}{b} = \frac{a}{b} + c.$$

Thus the equilibrium price decreases with b. This dovetails nicely with intuition: When the products are closer substitutes there is more competition in the market, which makes the firms choose lower prices.

(c) Find the Nash equilibrium when the firms do not have identical marginal costs.

Solution: Now the conditions for a Nash Equilibrium are:

$$p_1^* = \frac{a + bp_2^* + bc_1}{2b} \text{ and } p_2^* = \frac{a + bp_1^* + bc_2}{2b}.$$

Solve these two equations to get:

$$p_1^* = \frac{a}{b} + \frac{2}{3}c_1 + \frac{1}{3}c_2 \text{ and } p_2^* = \frac{a}{b} + \frac{2}{3}c_2 + \frac{1}{3}c_1.$$

(d) Suppose the marginal cost of firm j increases. What is the impact on the equilibrium price of firm i? Give an intuitive explanantion.

Solution: From the results above we see that p_i^* increases with c_j. When the cost of firm j increases, this makes firm j set a higher price. This allows firm i to set a higher price as well.

For the final question, let $a = b = 2$ and $c_1 = c_2 = 1$.

(e) Suppose the game between the two firms is repeated over an infinite time horizon $t = 1, 2, ..., \infty$. The discount factor is $\delta \in (0, 1)$ and each firm maximizes the sum of discounted profits. In this infinitely repeated game, specify trigger strategies such that the prices in all stages are $p_1 = p_2 = 4$. Find the smallest value of δ such that the trigger strategies constitute a subgame perfect Nash equilibrium.

Solution: Trigger strategy for firm i (note that with $a = b = 2$ and $c_1 = c_2 = 1$ the one-shot NE is $(p_1, p_2) = (2, 2)$):

- If $t = 1$ or the outcome of each previous stage has been $(p_1, p_2) = (4, 4)$, choose $p_i = 4$
- Otherwise, choose $p_i = 2$

With these trigger strategies, the outcome of all stages will be $(p_1, p_2) = (4, 4)$. The trigger strategies constitute a SPNE in the infinitely repeated game if it is not a profitable deviation for firm

i to play the one-shot best response to $p_j = 4$ and then the one-shot NE strategy ($p_i = 2$) in all future stages. The one-shot best response to $p_j = 4$ is (use the best response function derived earlier):

$$p_i = \frac{2 + 2 \cdot 4 + 2 \cdot 1}{2 \cdot 2} = 3.$$

Thus the sum of discounted profits from the deviation is

$$\pi_i(3,4) + \sum_{t=2}^{\infty} \delta^{t-1} \pi_i(2,2) = 8 + \sum_{t=2}^{\infty} \delta^{t-1} \cdot 2 = 8 + \frac{2\delta}{1-\delta}.$$

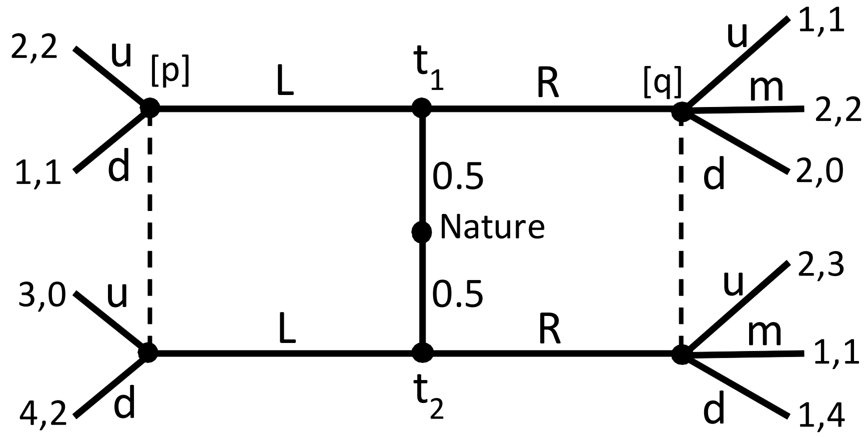
The sum of discounted profits from the trigger strategy is:

$$\sum_{t=1}^{\infty} \delta^{t-1} \pi_i(4,4) = \sum_{t=1}^{\infty} \delta^{t-1} \cdot 6 = \frac{6}{1-\delta}.$$

Thus the trigger strategies constitute a SPNE precisely if

$$\frac{6}{1-\delta} \geq 8 + \frac{2\delta}{1-\delta} \iff \delta \geq \frac{1}{3}.$$

3. Consider the following signalling game:



(a) Are any of the messages (L or R) dominated for type t_1? Are any of the messages dominated for type t_2?

Solution: None of the messages are dominated for t_1. For t_2, R is dominated by L (playing L gives a payoff of at least 3, playing R gives a payoff of at most 2).

(b) Find a separating perfect Bayesian equilibrium.

Solution: From the answer to (a) we know that the sender must play (R, L). Then the beliefs of the receiver must be $p = 0$ and $q = 1$. And then it follows that the receiver will play d after seeing the message L and m after seeing the message R. Finally, it is then easy to check that the original sender strategy is indeed optimal. Thus the only separating PBE is:

$$\{(R, L), (d, m), p = 0, q = 1\}.$$

(c) Find a pooling perfect Bayesian equilibrium. Does it satisfy Signalling Requirement 5 from Gibbons? Does it satisfy Signalling Requirement 6?

Solution: From the answer to (a) we know that the sender must play (L, L). Then the belief of the receiver after seeing L must be $p = \frac{1}{2}$. Therefore the receiver will play d after seeing L. After seeing the message R, the receiver must play u, otherwise t_1 will deviate from L (t_2 will never deviate because R is dominated by L). Thus q must be chosen such that it is optimal for the receiver to play u after seeing R. This means that the following inequalities must be satisfied:

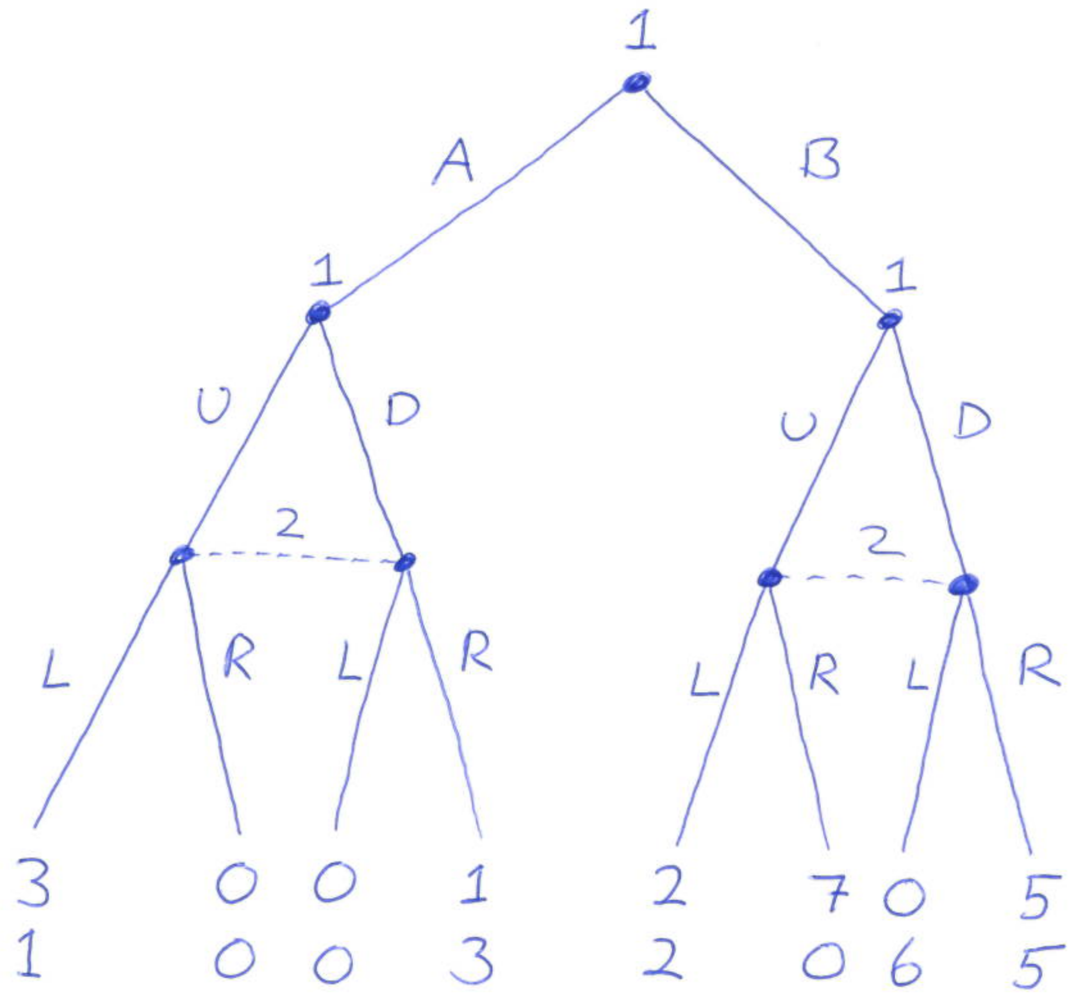
$$\begin{aligned} q + 3(1-q) &\geq 2q + (1-q) \text{ (u is better than m);} \\ q + 3(1-q) &\geq 4(1-q) \text{ (u is better than d).} \end{aligned}$$

From these inequalities we get $\frac{1}{2} \leq q \leq \frac{2}{3}$. Thus

$$\{(L, L), (d, u), p = \frac{1}{2}, q\}$$

is a pooling PBE for all $q \in [\frac{1}{2}, \frac{2}{3}]$ and there are no other pooling PBE.

Since $q < 1$ and R is dominated for t_1, in each of the pooling PBE above the receiver puts a positive probability on t_2 choosing a dominated message. Thus none of the pooling PBE satisfy Signalling Requirement 5. And since SR6 is a stronger requirement, it also follows that none of the pooling PBE satisfy SR6.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